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Damped FE model updating using complex updating parameters: Its use for dynamic design

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Abstract

Model updating techniques are used to update the finite element model of a structure so that updated model predicts the dynamics of a structure more accurately. The application of such an updated model in dynamic design demands that it also predicts the effects of structural modifications with a reasonable accuracy. Most of the model updating techniques neglect damping and so these updated models cannot be used for accurate prediction of complex frequency response functions (FRFs) and complex mode shapes. This paper deals with the basic formulation for the complex parameter based updating method and its use for dynamic design. A case involving actual measured data for the case of F-shaped test structure, which resembles the skeleton of a drilling machine is used to evaluate the effectiveness of complex parameter based updating method for accurate prediction of the complex FRFs. Structural modifications in terms of mass and beam modifications are introduced to evaluate the complex parameter based damped updated model for its usefulness in dynamic design.

1. Introduction

An accurate dynamic mathematical model of a structure is essential for predicting reliably its dynamic characteristics. Such a model allows improving the dynamic design of a structure at computer level resulting in an optimized design apart from savings in terms of money and time. It is well known that a mathematical (Finite element) model will be erroneous due to inevitable difficulties in modeling of joints, boundary conditions and damping. The experimental data are generally considered to be more accurate. The experimental approach to extract a model also faces problems due to limited number of measured co-ordinates, limited frequency range and difficulty in measurement of rotational degrees of freedom [1,2]. This has led to the development of model updating which aims at reducing the inaccuracies present in the analytical model in the light of measured dynamic test data. A number of model updating methods have been proposed in recent years as shown in the surveys by Imregun and Visser [3] and Mottershead and Friswell [4] and details of these can be found in the text by Friswell and Mottershead [5]. A significant number of methods [6–8],

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which were first to emerge belonged to the direct category. These methods violate structural connectivity and matrices are difficult to interpret. On the other hand, iterative methods provide wide choices of updating parameters, structural connectivity can be easily maintained and corrections suggested in the selected parameters can be physically interpreted. Iterative methods either use eigendata or FRF (frequency response function) data. Collins et al. [9] used the eigendata sensitivity for model updating in an iterative framework. Lin and Ewins [10] used measured FRF data to update an analytical model. Most of the updating methods neglect the damping. These cannot be used for predicting complex FRFs and damping. All structures exhibit some form of damping, but despite a large literature on damping, it still remains one of the least well-understood aspects of general vibration analysis. A commonly used model originated by Rayleigh [11] assumes that instantaneous generalized velocities are the only variables and damping matrix is directly analogous to the mass and stiffness matrices. Material damping can arise from a variety of micro structural mechanisms [12] but for small strains it is often adequate to represent it through an equivalent linear model of the material. Maia et al. [13] emphasized the need for the development of identification methodologies of general damping models and indicated several difficulties that might arise. Pilkey [14] describes two types of procedures, direct and iterative, for computation of the system-damping matrix. Adhikari and Woodhouse [15] identified the damping of the system as viscous damping. However, viscous damping is by no means the only damping model. Adhikari and Woodhouse [16] identified non-viscous damping model using an exponentially decaying relaxation function.

Some research efforts have also been made to update the damping matrices. Imregun et al. [17,18] conducted several studies using simulated and experimental data to gauge the effectiveness of RFM and extended the RFM to update proportional damping matrix by updating the coefficients of damping matrix. Yong and Zhenguo [19] proposed a two step model updating procedure for lightly damped structures using neural networks. In the first step, mass and stiffness are updated using natural and antiresonance frequencies. In the second step, damping ratios are updated. Lin and Zhu [20] extended RFM to update damping coefficients in addition to mass and stiffness matrices. Arora et al. [21] proposed a complex parameter based model updating method in which FE model is updated in such a way that the updated model reflects general damping in the experimental model by considering the updating parameters as complex.

A model updating method should able to predict the changes in dynamic characteristics of the structure due to potential structural modifications. Very little appears to have been done from this aspect though there is lot of work reported on FE model updating itself. Modak et al. [22] compared predictions of dynamic characteristics using undamped updated finite element models. Arora et al. [23] proposed a damped FE model updating procedure and studied its effectiveness for dynamic design. This paper deals with the basic formulation for the complex parameter based updating method to obtain damped updated model and its use for dynamic design. A case involving actual measured data for the case of F-shaped test structure, which resembles the skeleton of a drilling machine is used to evaluate the effectiveness of complex parameter based updating method for accurate prediction of the complex FRFs. Thereafter damped updated model is utilized for predicting the effects of structural modifications. Structural modifications, one in the form of a lumped mass and another in the form of beam, are introduced to check the damped updated FE model for its usefulness in dynamic design.

2. Basic theory

Complex parameter based updating method [21] is a development of Response function method given by Lin and Ewins [10], which is an iterative method and uses measured FRF data directly without requiring any modal extraction. In this method, it is assumed that initially there is no damping in the analytical model, which results in real FRFs where as experimental FRFs are complex because of the presence of damping. The following identities relating dynamic stiffness matrix $[Z]$ and receptance FRF matrix $[\alpha]$ can be written for the analytical model as well as for the actual structure, respectively,

$$[Z_A^R][\alpha_A^R] = [I] \quad \text{(Initially)} \tag{1}$$

$$[Z_X^C][\alpha_X^C] = [I] \tag{2}$$

where subscripts A and X denote analytical (like an FE model) and experimental model, respectively, and superscripts R and C represent real and complex values. Expressing $[Z_X]$ in Eq. (2) as $[Z_A^R]+[\Delta Z]$ and then subtracting Eq. (1) from it, following matrix equation is obtained

$$[\Delta Z^R][\alpha_X^C] = [Z_A^R]([\alpha_A^R] - [\alpha_X^C]) \tag{3}$$

Pre-multiplying above equation by $[\alpha_A^R]$ and using Eq. (1) gives

$$[\alpha_A^R][\Delta Z^R][\alpha_X^C] = [\alpha_A^R] - [\alpha_X^C] \tag{4}$$

If only the jth column of experimentally measured FRF matrix $\{\alpha_X^C\}_j$, is available then the above equation is reduced to

$$[\alpha_A^R][\Delta Z^R]\{\alpha_X^C\}_j = \{\alpha_A^R\}_j - \{\alpha_X^C\}_j \tag{5}$$

which is the basic relationship of the response function method. With this method, a physical variables based updating parameter formulation is used. Linearizing $[\Delta Z]$ with respect to $\{p\}$, $\{p\} = \{p_1, p_2, \ldots, p_{\mathrm{nu}}\}^{\mathrm{t}}$ being the vector of physical variables associated with individual or group of finite elements, gives

$$[\Delta Z] = \sum_{i=1}^{\mathrm{nu}} \left(\frac{\partial [Z]}{\partial p_i} \cdot \Delta p_i \right) \tag{6}$$

nu is the total number of updating parameters. $[Z]$ in Eq. (6) is replaced by $[K]-\omega^2[M]$. Dividing and multiplying Eq. (6) by p_i and then writing u_i in place of $\Delta p_i/p_i$. We can write Eq. (6) as

$$[\Delta Z] = \sum_{i=1}^{\mathrm{nu}} \left(\frac{\partial([K]-\omega^2[M])}{\partial p_i} \cdot p_i \right) \cdot u_i \tag{7}$$

Thus $\{u\} = \{u_1, u_2, \ldots, u_{\mathrm{nu}}\}^{\mathrm{t}}$ is the unknown vector of fractional correction factors to be determined during updating. Eq. (5), after making the substitution for $[\Delta Z]$ from Eq. (7), can be written at various frequency points chosen from the frequency range considered. The frequency points are chosen on the criteria that the points should lie away from the resonance and anti-resonance frequencies [24]. The resulting equations can be framed in the following matrix form:

$$[S(\omega)^C]_{(n\times \mathrm{nf})\times(\mathrm{nu})}\{u^C\}_{\mathrm{nu}\times 1} = \{\Delta\alpha(\omega)^C\}_{(n\times \mathrm{nf})\times 1} \tag{8}$$

Here $[S]$ is sensitivity matrix and $\{\Delta\alpha\}$ is the difference between analytical and experimental FRFs values at the selected frequency points. The fractional correction factors $\{u\}$ obtained using Eq. (8) are complex. As correction factors are complex this results in complex updating parameters. The updated version of analytical finite element model is built using this set of complex parameters of physical variables. The parameter vector of physical variables $\{p\}$ is considered to be complex in the form

$$\{p^C\} = \{p^R\} + j\{p^I\} \tag{9}$$

The real part of the complex updating parameter represents change in physical variable. The imaginary part relates to damping of the system. The process is repeated in an iterative way until convergence is obtained. At the end of first iteration, correction factor would have an updated physical variable given by

$$p_1^C = (1 + u_1^C)p_0^R \tag{10}$$

Initially, the parameter of the physical variable is real and after first iteration the updated parameter becomes complex. Similarly at the end of second iteration the updated physical variable value will be

$$p_2^C = (1 + u_2^C)p_1^C \tag{11}$$

The performance is judged on the basis of the accuracy with which the FRFs predicted by updated FE model match with the simulated experimental FRFs or the experimental FRFs.

3. Structural modification using damped updated FE model

Damped updated finite element model for a structure is available in terms of complex stiffness mass matrices denoted by $[K^C]$ and $[M^C]$, respectively. The complex stiffness and mass matrices can be written as

$$[K^C] = [K^R] + j[K^I] \tag{12}$$

$$[M^C] = [M^R] + j[M^I] \tag{13}$$

In design practice, the size of modifications is very small as compared to the structure. It is assumed that there is no effect of design modifications on the damping of the structure. The structural modifications will affect the real part of complex mass and stiffness matrices. If $[\Delta K^R]$ and $[\Delta M^R]$ represent the modification matrices due to structural modification then the modified structure's real part of stiffness and mass matrices are denoted by $[K_m^R]$ and $[M_m^R]$, respectively, and can be written as

$$[K_m^R] = [K^R] + [\Delta K^R] \tag{14}$$

$$[M_m^R] = [M^R] + [\Delta M^R] \tag{15}$$

The modified real part of stiffness and mass matrices are using to generate modified complex stiffness and mass matrices as

$$[K_m^C] = [K_m^R] + j[K^I] \tag{16}$$

$$[M_m^C] = [M_m^R] + j[M^I] \tag{17}$$

These modified complex mass and stiffness matrices are then used for predicting the effects of structural modifications. Consider the case of mass modification by assuming that a mass m_0 kg is added at the ith node. The $[\Delta M^R]$ is obtained by making the diagonal entries corresponding to the translational degrees of freedom for the ith node equal to '$+m_0$' assuming that the rotary inertia of the modification is negligible [25]. For the case of beam modification the $[K_m^R]$ and $[M_m^R]$ are essentially obtained by assembling the FE-model for the modified beam member.

4. Damped FE model updating of F-shaped structure

The complex parameter based updating method is evaluated for the case of an F-shaped structure, as shown in Fig. 1, using experimental data. The F-shape structure has been constructed by bolting the two beam

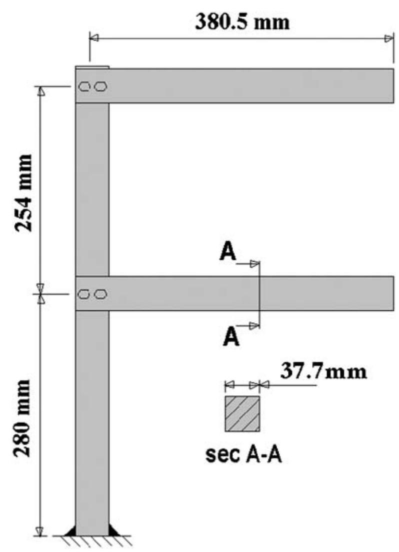

Fig. 1. F-shaped structure.

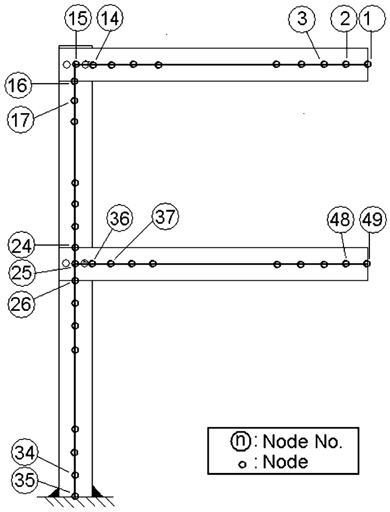

Fig. 2. Initial FE model.

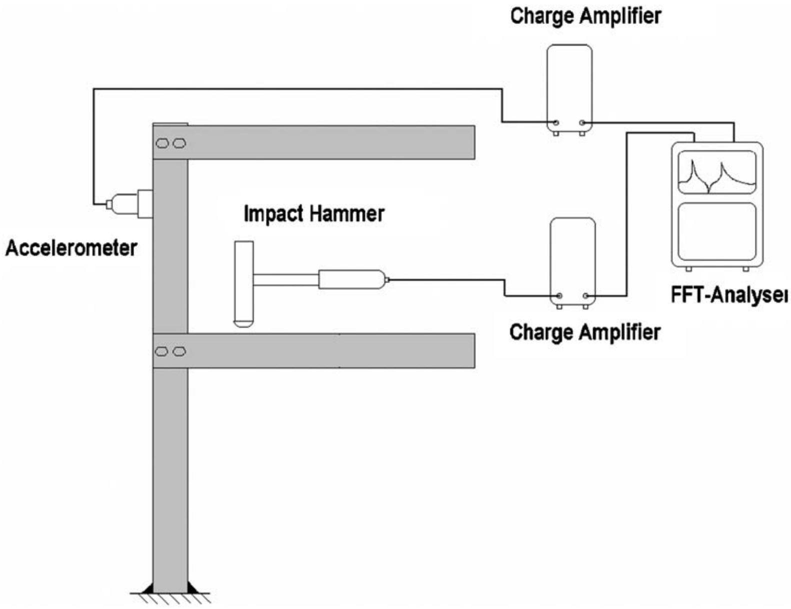

Fig. 3. Instrumentation set-up for modal test using impact excitation.

Table 1
Correlation of measured and FE-model based modal data of F-shaped structure before updating.

Mode no.	Measured frequency (Hz)	FE-model predictions		MAC-value
		Frequency (Hz)	% Error	
1	34.95	43.05	23.17	0.9650
2	104.02	123.67	18.89	0.9364
3	133.96	185.21	38.26	0.9311
4	317.52	385.17	21.30	0.9141
5	980.16	1020.06	4.07	0.6908

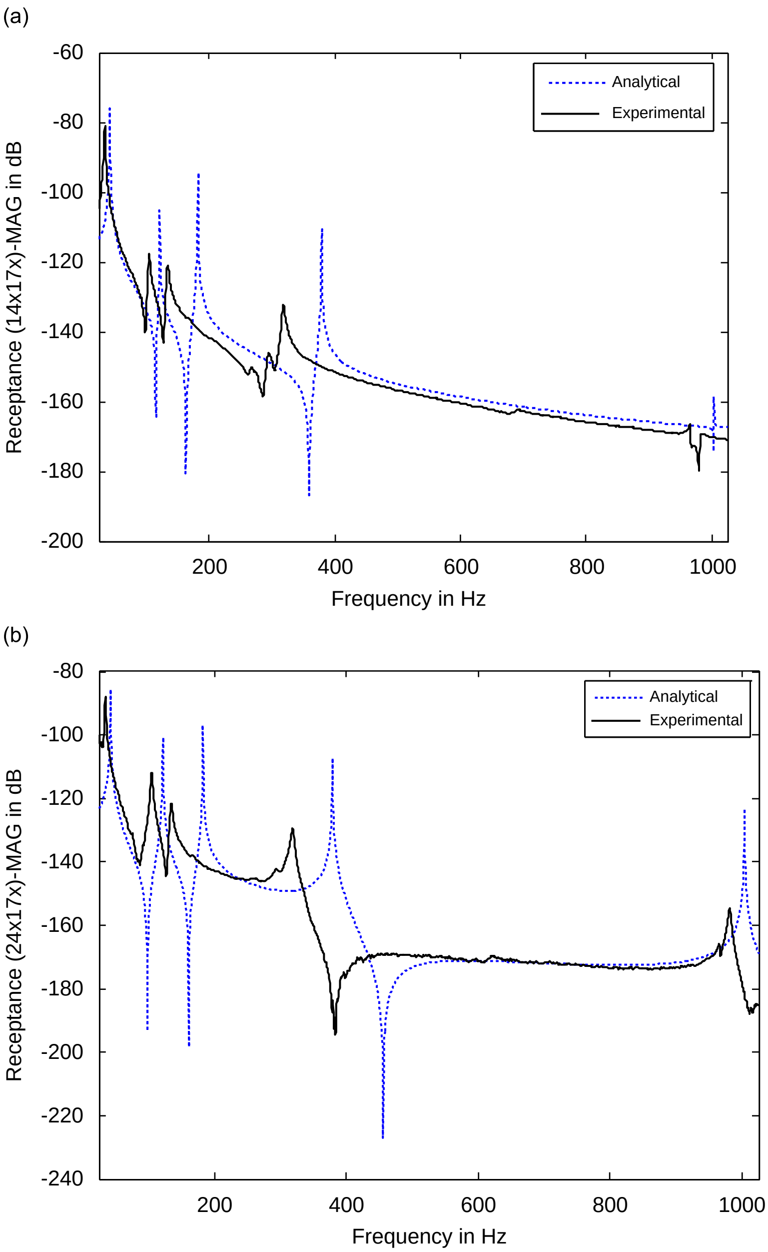

Fig. 4. Overlay of the measured FRFs and the corresponding FE model FRFs before updating.

Table 2
Values of torsional springs stiffness of each joint after updating of the F-shaped structure.

Updating variable	Initial value (N mrad^{-1})	Updated values using proposed method (N mrad^{-1})	
		Real	Imaginary
K_{t1}	3.28E+06	2.59E+05	4.36E+03
K_{t2}	3.28E+06	2.78E+05	5.1E+03
K_{t3}	3.28E+06	3.1E+05	5.78E+03

Table 3
Correlation between the measured and updated model.

Mode no.	Measured frequency (Hz)	Updated model predictions		
		Frequency (Hz)	% Error	MAC-value
1	34.95	34.25	−2.0	0.9923
2	104.02	100.27	−3.60	0.9693
3	133.96	134.42	0.34	0.9675
4	317.52	313.73	−1.19	0.9423
5	980.16	973.44	−0.68	0.4370

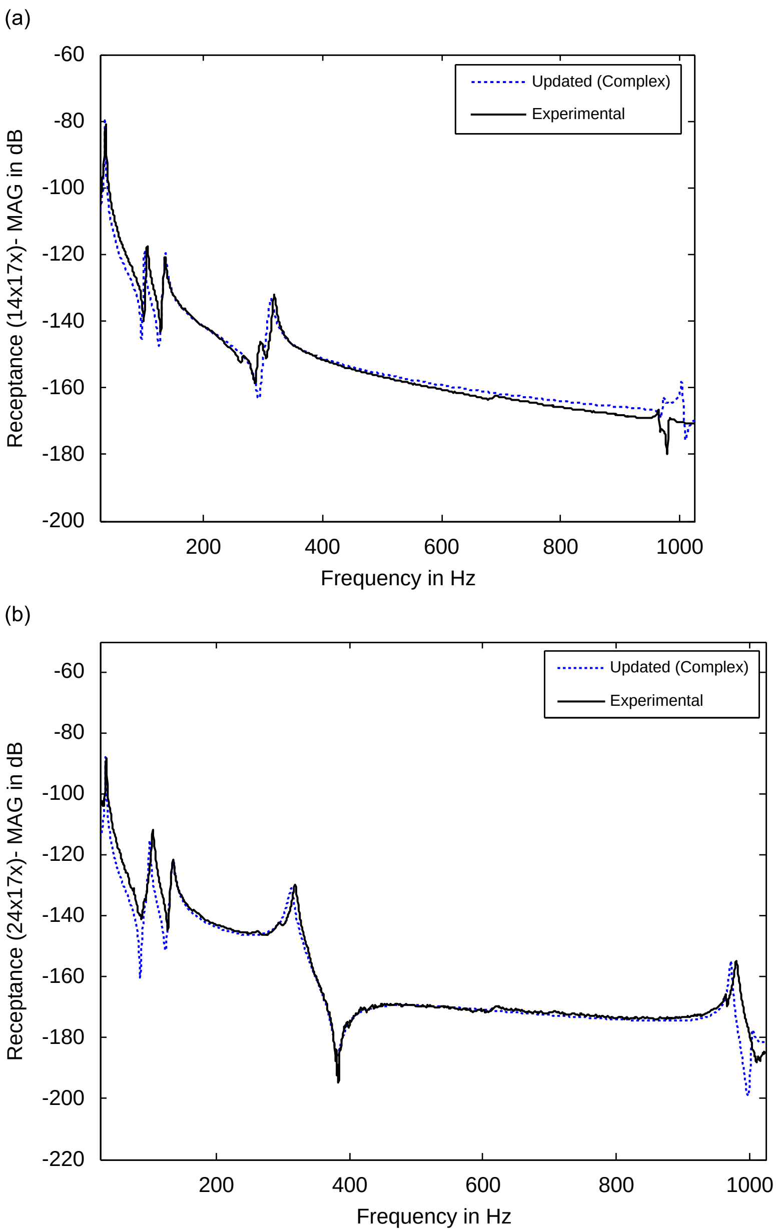

Fig. 5. Overlay of the measured FRFs and the corresponding updated model FRFs after complex model updating.

members horizontally to a vertical beam member, which in turn, has been welded to the base plate at the bottom. All the beam members have a square cross-section with 37.7 mm side. A Finite element model of the F-structure is built, as shown in Fig. 2, using 48 two-dimensional frame elements (two translational degrees of freedom in x and y direction and one rotational degree of freedom, per node) to model in-plane dynamics. In the F-shaped structure, there are three joints, which are modeled by taking coincident nodes at each of them. Thus the, two nodes which are geometrically coincident are taken as one joint instead of one node. A horizontal, a vertical and a torsional spring couple the two nodes at each of such a coincident pair. The stiffnesses of these springs are K_x, K_y and K_t, respectively. The modal test is performed by exciting the structure with an impact hammer at 16 locations and response is measured at one location using accelerometer

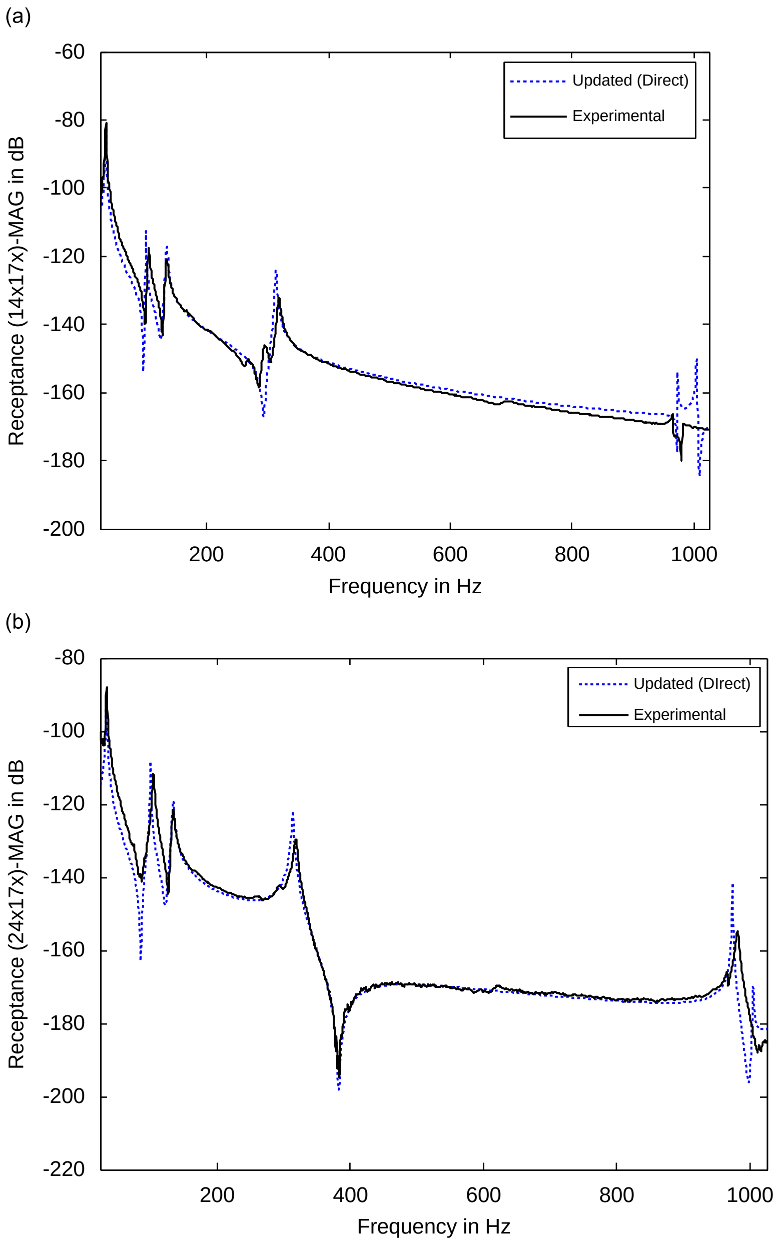

Fig. 6. Overlay of the measured FRFs and the corresponding updated model FRFs after updating using FE model updating with damping identification method.

as shown in Fig. 3. The FRFs so acquired are analyzed using a global curve fitting technique available in ICATS [26] to obtain experimental sets of modes in the range of 0–1000 Hz.

The correlation between the analytical and the experimental set of modal data is now performed using MAC which is calculated for the pair of ith analytical and jth measured mode shape as

$$\mathrm{MAC}(\{\phi_A\}^i, \{\phi_X\}^j) = \frac{(\{\phi_A\}^{iT}\{\phi_X\}^j)^2}{(\{\phi_A\}^{iT}\{\phi_A\}^j)(\{\phi_X\}^{iT}\{\phi_X\}^j)} \tag{18}$$

A comparison of the corresponding experimental and analytical natural frequencies, the percentage difference between them and the corresponding MAC-value for first five modes are given in Table 1. An overlay of the measured FRFs and the corresponding FE model FRFs are shown in Fig. 4. The FRF 14×17x represents excitation at node 14 and response at node 17 both in x-direction. It is observed that the shape of the FE model FRF-curve is similar to the measured curve. It therefore infers that though the FE model is in error it is, in principle, of updatable quality.

Choice of updating parameters on the basis of engineering judgment about the possible locations of modeling errors in a structure is one of the strategies to ensure that only physical meaningful corrections are made. In case of F-structure, modeling of stiffness of the joints is expected to be a dominant source of inaccuracy in the FE model assuming that the values of material and the geometric parameters are correctly known. Analytical sensitivity analysis of the joint springs shows that the torsional stiffness is the most important variable affecting the natural frequencies. Torsional springs of stiffness K_{t1}, K_{t2} and K_{t3} coupling the rotational degrees of freedom of the coincident nodes at the three joints are taken as updating parameters. The other two degrees of freedom of the coincident nodes are taken as rigidly coupled.

The initial and final values of the torsional spring stiffness of each joint are given in Table 2. It is observed that the real values of stiffness of the torsional springs corresponding to three joints are reduced and also values of three springs are not very different from each other while the imaginary values of the stiffness represent damping in the system. The frequency points selected on the criteria that the points should lie away from the resonance and anti-resonance frequencies [24]. The frequency points considered for updating of F-shape structure are 27, 31, 99, 101, 131, 311,442, 515, 825 Hz for each of the FRFs. A comparison of the correlation between the measured and the updated model natural frequencies is given in Table 3. It is observed from Table 3 that for the complex parameter based updating method there is a significant reduction in the error in natural frequencies. Fig. 5 shows the overlay of measured and updated FRFs. It is noticed that the shape of the updated FRFs is same as that measured FRFs. The results of the complex parameter based updating are compared with FE model updating with damping identification method proposed by Arora et al. [23]. In this method, damped updated model is obtained in two steps. In the first step, mass and stiffness matrices are updated and in the second step, damping is identified using updated mass and stiffness matrices, obtained in previous step. The overlay of the measured and the updating FRFs obtained by FE model

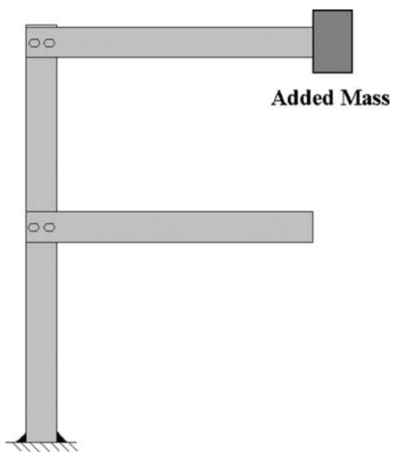

Fig. 7. F-shape-structure with mass modification.

updating with damping identification are shown in Fig. 6. It can be noticed from Figs. 5 and 6 that complex parameter based updating method is able to predict more accurately FRFs as compared to FE model updating with damping identification method.

5. Structural dynamic modification using damped updated FE model

The damped updated model obtained above is used for predicting the effects of potential design modifications made to the structure. This section gives a comparison of the measured changes in dynamic

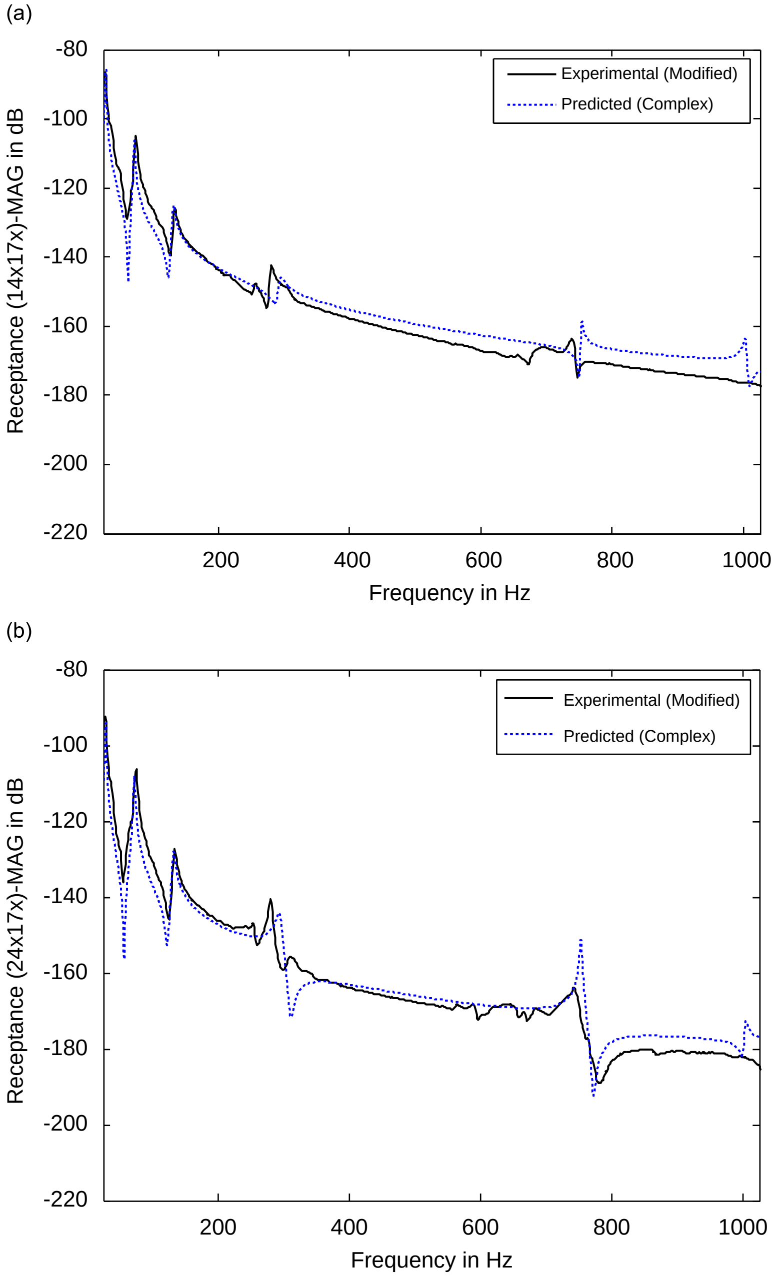

Fig. 8. Overlay of the measured FRFs and the corresponding predicted FRFs after mass modification using complex updating parameters method.

Table 4
Comparison of the dynamic characteristics for the case of mass modification.

Mode no.	Measured frequency (Hz)	FE-model predictions		Damped updated model predictions	
		Frequency (Hz)	% Error	Frequency (Hz)	% Error
1	27.32	34.93	−27.85	28.45	−4.13
2	74.53	91.49	−22.75	72.38	2.87
3	133.38	178.97	−34.18	131.58	1.34
4	280.11	357.51	−27.63	293.65	−4.83

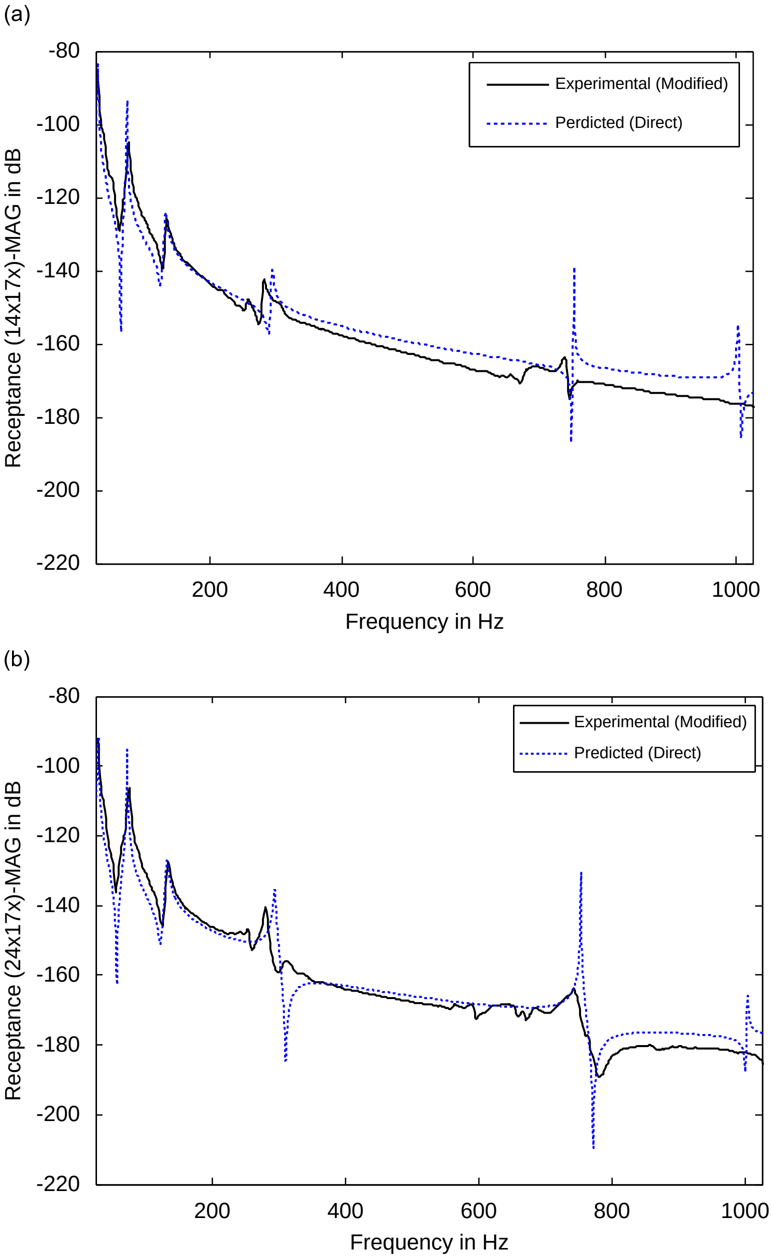

Fig. 9. Overlay of the measured FRFs and the corresponding predicted FRFs after mass modification by FE model updating with damping identification method.

characteristics due to structural modifications with those predicted using the updated model. The comparison is performed first for a mass modification and then for a beam modification.

5.1. Mass modification

A mass modification is introduced by attaching a mass of 1.8 kg at the tip of the upper horizontal beam member as shown in Fig. 7. The FRFs for the mass-modified structure are then acquired. The mass modification is also introduced analytically in the damped updated model. The mass matrix for the modified structure, and subsequently its modal data and FRFs, corresponding to the updated model are obtained. A comparison of the modified FRF as predicted by complex parameter based updating method is shown in Fig. 8 while a comparison of natural frequencies is given in Table 4. It is observed from Fig. 8 and Table 4 that the predicted dynamic characteristics of complex parameter based updated model are closer to the measured characteristics of the modified structure even at resonance and anti-resonance frequencies. The average percentage error in the predictions for the first four natural frequencies based on the FE-model is 28.1 percent while that based on the damped updated model is much less at 3.29 percent. The predictions of complex parameter based updating method for mass modification are compared with FE model updating with damping identification method [23]. The overlay of the measured and the predicted FRFs after mass modification by FE model updating with damping identification are shown in Fig. 9. It can be noticed from Figs. 8 and 9 that complex parameter based updating method is able to predict more accurately amplitudes of vibration at resonance points as compared to model updating with damping identification.

5.2. Beam modification

A beam modification is introduced in the form of a stiffener of width 38.2 mm and thickness 5 mm. The stiffener is attached between the tips of the lower and the upper horizontal beam members as shown in Fig. 10. The beam is connected to the F-structure by bolted joints. The FRFs for the beam-modified structure are then acquired. The beam modification is also introduced analytically in the updated with identified damping model. The beam modification will increase the size of mass and stiffness matrices. The mass and stiffness matrices for the modified structure are obtained assuming there is little effect of the beam modification on the damping of the system, and subsequently its modal data and FRFs, corresponding to the updated model are obtained. The overlay of the modified FRF as predicted by complex parameter based updating model and measured modified FRF is shown in Fig. 11 while a comparison of dynamic characteristics predicted by complex parameter based updated model and FE model is given in Table 5. It is observed from Fig. 11 and Table 5 that

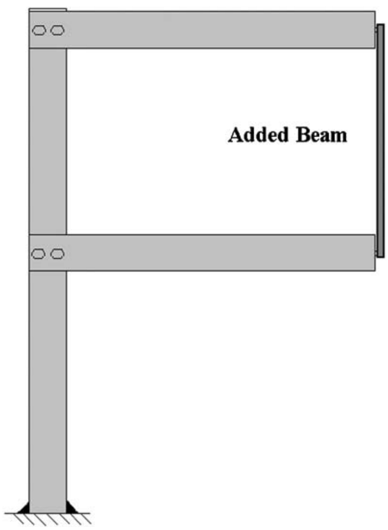

Fig. 10. F-shape-structure with beam modification.

the dynamic characteristics predicted by complex parameter based updated model are closer to the measured characteristics of the modified structure. The average percentage error in the predictions for the first four natural frequencies based on the FE-model is −23.69 percent while that based on the damped updated model is much less at 3.0 percent. The predictions of complex parameter based updating method for beam modification are also compared with FE model updating with damping identification method [23]. The overlay of the measured and the predicted FRFs after beam modification by FE model updating with damping identification method are shown in Fig. 12. It can be noticed from Figs. 11 and 12 that complex parameter based updating method is able to predict more accurately.

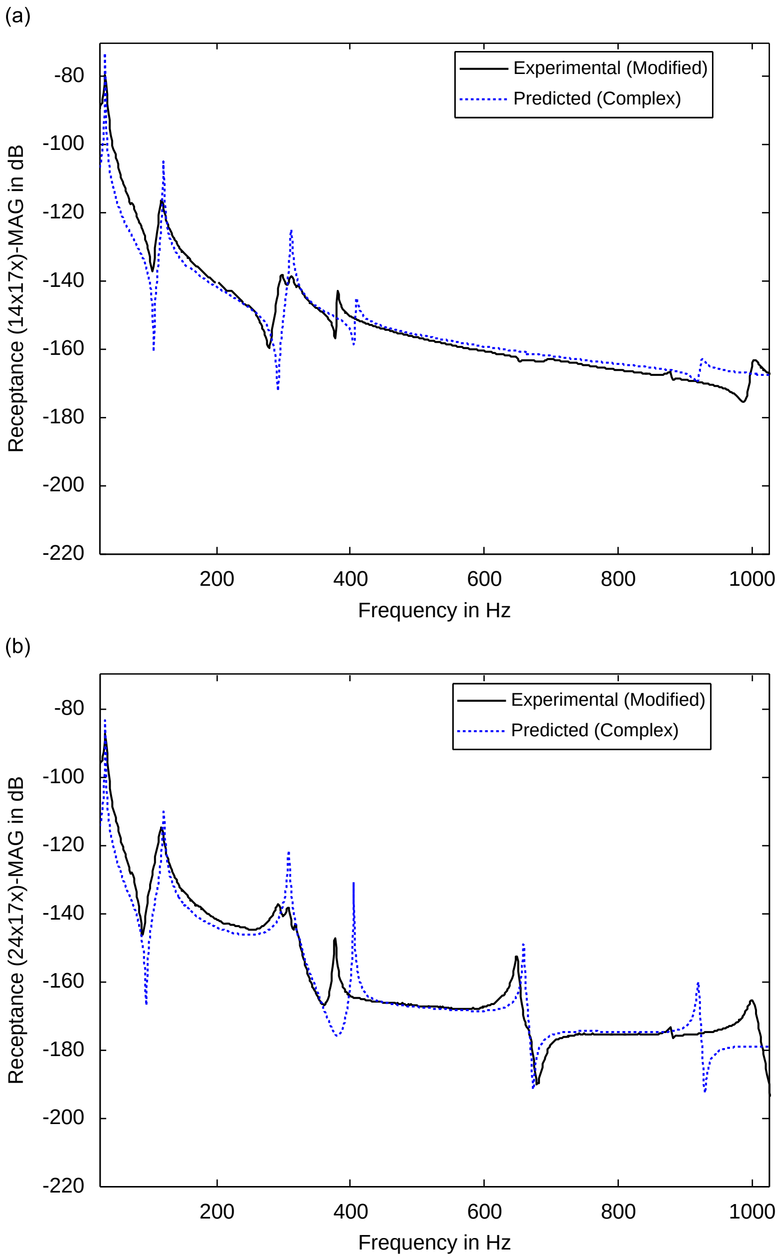

Fig. 11. Overlay of the measured FRF and the corresponding predicted FRF after beam modification using complex updating parameters method.

Table 5
Comparison of the dynamic characteristics for the case of beam modification.

Mode no.	Measured frequency (Hz)	FE-model predictions		Damped updated model predictions	
		Frequency (Hz)	% Error	Frequency (Hz)	% Error
1	33.95	42.91	−26.39	33.66	0.85
2	117.30	165.14	−40.78	120.75	−2.94
3	309.98	371.97	−19.99	307.78	0.71
4	376.89	405.56	−7.60	405.23	−7.52

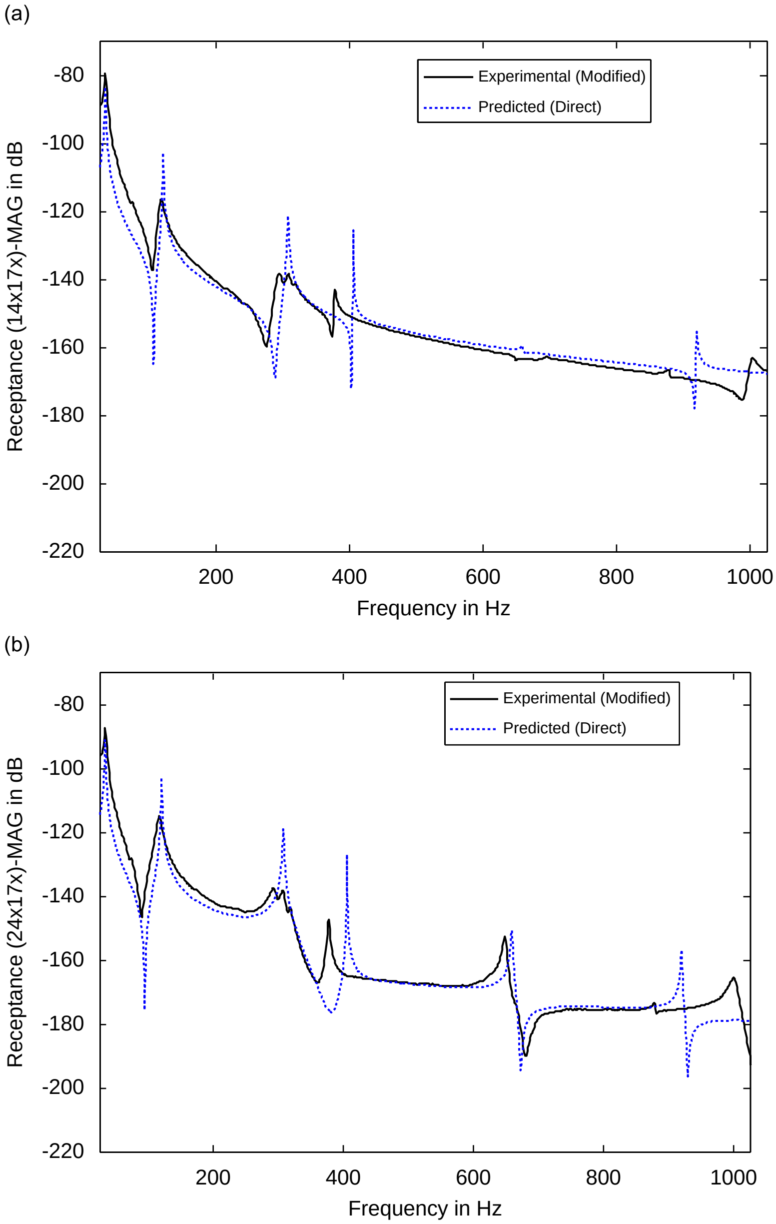

Fig. 12. Overlay of the measured FRFs and the corresponding predicted FRFs after beam modification by FE model updating with damping identification method.

It can also be noticed from Figs. 9 and 11 that predicted FRFs for mass modifications matches better than beam modification. For beam modification no estimation is carried out for the damping and stiffness of the joints. In this study, the focus is to evaluate the prediction capabilities of the complex parameter based updated model and it can be concluded with confidence that complex parameter based updated model can be used for dynamic design.

6. Conclusions

In this paper a damped FE model for a complex structure has been employed for performing dynamic design. Damped updated FE-model for an F-shape structure is obtained by using complex parameter based updating method. The dynamic design at the computer level has been demonstrated via mass and beam stiffener using damped updated FE model using complex updating parameters. It is seen that damped updated FE model predicts accurately not only the natural frequencies but also complex FRFs. The modified dynamic characteristics due to modifications obtained via damped updated FE-model indicate, on experimental verification, that they are of acceptable accuracy. Thus it can accordingly be concluded that damped updated FE model obtained by complex parameters can be used for dynamic design with confidence.

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